



# **TExES Mathematics 4-8 (115) Exam**

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- These slides overview information for each domain within the Math 4-8 (115) Exam. This test is designed to assess whether an examinee has the requisite knowledge and skills that an entry-level educator in this field in Texas public schools must possess.
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# TutoringEZ



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# Math 4–8 Test Overview

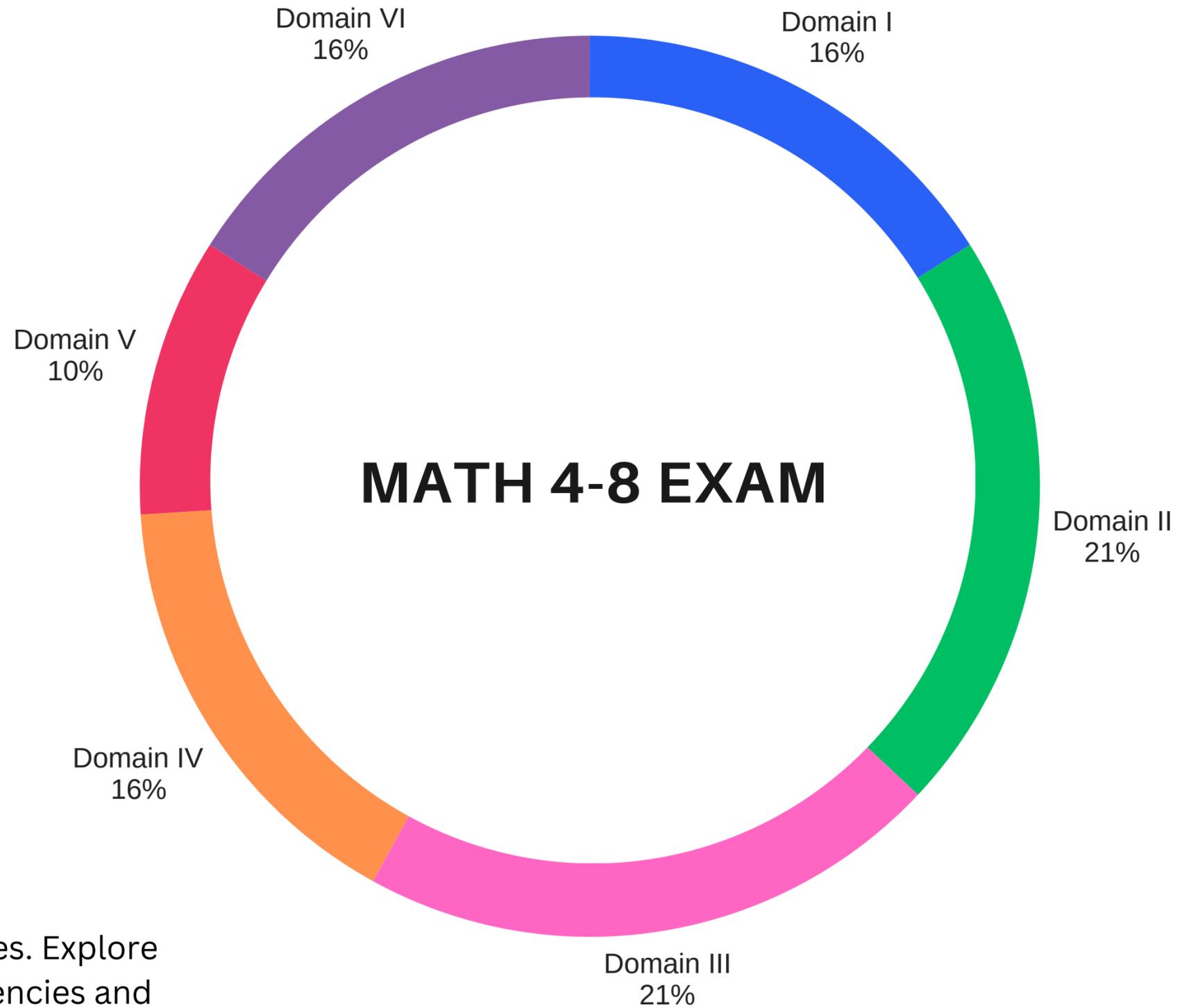
**01.** Domains I–VI

**02.** 5 hours

**03.** 100 selected-  
response questions



- I** **Number Concepts**
- II** **Patterns & Algebra**
- III** **Geometry & Measurement**
- IV** **Probability & Statistics**
- V** **Mathematical Processes & Perspectives**
- VI** **Mathematical Learning, Instruction, & Assessment**



Each section is made up of a number of competencies. Explore each section to learn the breakdown of the competencies and understand the type of questions in that section.

Domain I



# Number Concepts



# Domain I Competencies

1.

NUMBER SYSTEMS

2.

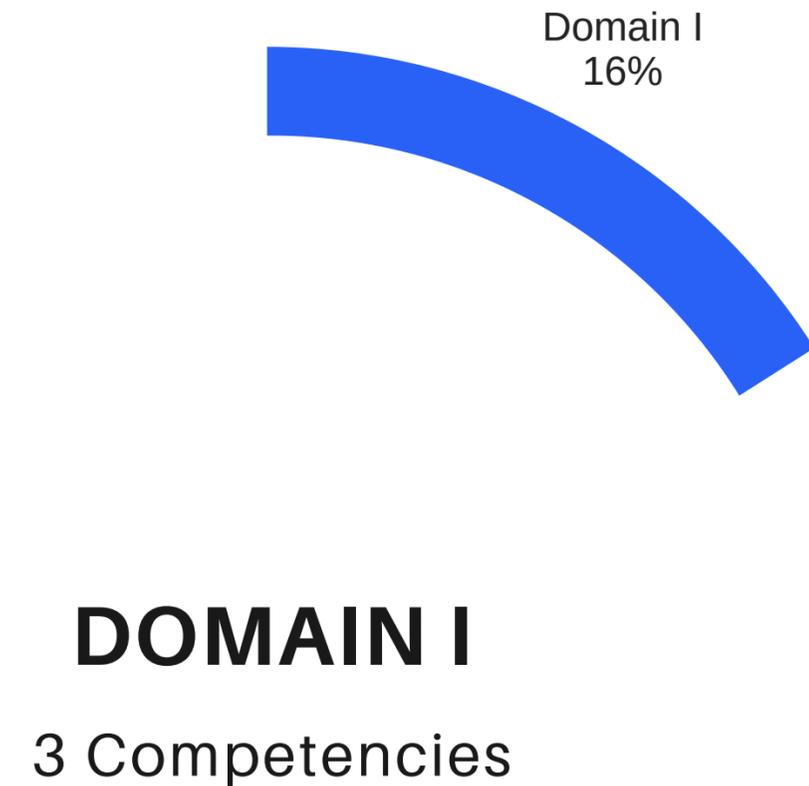
OPERATIONS AND  
ALGORITHMS

3.

NUMBER THEORY

## Number Concepts

Domain I focuses on understanding number systems, performing operations with computational algorithms, and applying number theory to solve mathematical problems.



# Number Systems

The teacher understands the structure of number systems, the development of a sense of quantity and the relationship between quantity and symbolic representations.

## Overview of Competency 1

- Understand place value and the structure of numeration systems
- Compare relative magnitude across whole numbers, integers, rationals, and real numbers
- Use multiple models to represent numbers (fraction strips, number lines, diagrams)
- Convert between equivalent representations (fractions, decimals, percents, scientific notation)
- Recognize number system properties (closure, commutativity, identity, inverse elements)



## Sample Question for Competency 1



Which of the following statements best explains why the equation  $x^2 = -16$  has no solution in the real number system but does have a solution in the complex number system?

- A.** Real numbers cannot be negative, while complex numbers can be negative.
- B.** The real number system does not include square roots, while the complex number system does.
- C.** The square of any real number is non-negative, while complex numbers include imaginary units that allow for negative squares.
- D.** Real numbers are closed under multiplication, while complex numbers are not.

# Number Systems

**Answer: C**



In the real number system, squaring any number (positive or negative) always produces a non-negative result. For example,  $4^2 = 16$  and  $(-4)^2 = 16$ . Therefore,  $x^2 = -16$  has no real solution. However, the complex number system includes the imaginary unit  $i$ , where  $i^2 = -1$ . This allows us to express solutions like  $x = 4i$  or  $x = -4i$ , since  $(4i)^2 = 16i^2 = 16(-1) = -16$ . This demonstrates how some equations that are unsolvable in one number system have solutions when we extend to another number system.



# Operations and Algorithms

The teacher understands number operations and computational algorithms.



## Overview of Competency 2

- Perform operations proficiently with real and complex numbers
- Explain relationships between number properties, operations, and algorithms
- Use concrete and visual representations to connect operations and algorithms
- Justify computational procedures and identify common error patterns
- Extend operations from basic numbers to algebraic expressions (e.g., fractions to rational expressions)



# Operations and Algorithms

## Sample Question for Competency 2



A student adds the fractions  $\frac{2}{3} + \frac{3}{4}$  and gets  $\frac{5}{7}$  by adding the numerators and denominators separately. Which of the following best addresses the student's error pattern?

- A.** Have the student memorize the correct algorithm for adding fractions.
- B.** Use visual models like fraction strips to show that  $\frac{2}{3} + \frac{3}{4}$  represents combining parts of different-sized wholes.
- C.** Tell the student that fractions must have common denominators before adding.
- D.** Practice more problems until the student remembers the correct procedure.



# Operations and Algorithms

**Answer: B**



The student's error shows a conceptual misunderstanding of what fractions represent, not just a procedural mistake. Using concrete and visual representations (like fraction strips or area models) helps students understand why we need common denominators—because we can only add parts when they're the same size. By seeing that  $\frac{2}{3}$  uses thirds and  $\frac{3}{4}$  uses fourths (different-sized pieces), students grasp that we must first convert both fractions to the same-sized pieces (twelfths) before combining. This builds conceptual understanding that justifies the algorithm, rather than just memorizing steps without understanding.

# Number Theory

The teacher understands ideas of number theory and uses numbers to model and solve problems within and outside of mathematics.

## Overview of Competency 3



- Apply number theory concepts like prime factorization and greatest common divisor
- Use numbers to quantify real-world phenomena (money, length, area, volume, density)
- Develop mental math and estimation techniques using place value and number properties
- Apply counting techniques (permutations and combinations) to solve problems
- Use properties of real numbers in theoretical and applied contexts



## Sample Question for Competency 2



A school is organizing students into groups for a field trip. There are 48 sixth graders and 72 seventh graders. The teacher wants to create the largest possible groups where each group has the same number of sixth graders and the same number of seventh graders, with no students left over. How many students will be in each group?

- A. 12 students
- B. 20 students
- C. 24 students
- D. 5 students



# Number Theory

## Answer: D



This problem requires finding the greatest common divisor (GCD) of 48 and 72 to determine the maximum number of groups that can be formed.

First, find the prime factorization of each number:

- $48 = 2^4 \times 3$
- $72 = 2^3 \times 3^2$

The GCD is found by taking the lowest power of each common prime factor:  $2^3 \times 3 = 8 \times 3 = 24$ .

This means we can create 24 equal groups (the largest number possible).

To find how many students are in each group:

- Sixth graders per group:  $48 \div 24 = 2$  students
- Seventh graders per group:  $72 \div 24 = 3$  students
- Total per group:  $2 + 3 = 5$  students



This demonstrates a practical application of number theory where the GCD helps solve a real-world organizational problem, ensuring all students are included with no one left over.

Domain II



# Patterns and Algebra



# Domain II Competencies

4.

PATTERNS AND RELATIONS

5.

LINEAR FUNCTIONS

6.

NONLINEAR FUNCTIONS

7.

CALCULUS FOUNDATIONS

63%

## DOMAIN II

4 Competencies

Domain II  
21%

### Patterns and Algebra

This section has 4 competencies. Review the following slides to understand each competency on the test.

# Patterns and Relations

The teacher understands and uses mathematical reasoning to identify, extend and analyze patterns and understands the relationships among variables, expressions, equations, inequalities, relations and functions.



## Overview of Competency 4

- Use inductive reasoning to identify, extend, and create patterns with models, numbers, and expressions
- Formulate rules to describe sequences verbally, numerically, graphically, and symbolically
- Make and test conjectures about patterns and relationships in data
- Justify manipulation of algebraic expressions
- Illustrate functions using concrete models, tables, graphs, and symbolic representations
- Use transformations to explore properties of functions and relations

# Patterns and Relations

## Sample Question for Competency 4

A student observes the following pattern in a sequence of figures made with tiles:

- Figure 1: 4 tiles
- Figure 2: 7 tiles
- Figure 3: 10 tiles
- Figure 4: 13 tiles

Which explicit rule best represents the number of tiles  $T$  in figure  $n$ ?

A.  $T = 4n$

B.  $T = 3n + 1$

C.  $T = n + 3$

D.  $T = 4n - 3$



# Data-Driven Decision Making

## Answer: B

This problem requires identifying the pattern and creating an explicit rule. First, examine the differences between consecutive terms:  $7-4=3$ ,  $10-7=3$ ,  $13-10=3$ . The common difference of 3 indicates a linear pattern with a slope of 3.

To find the explicit formula:

- The pattern starts at 4 when  $n = 1$
- It increases by 3 each time
- Using the formula  $T = 3n + b$ , substitute:  $4 = 3(1) + b$ , so  $b = 1$
- Therefore:  $T = 3n + 1$

# Linear Functions

The teacher understands and uses linear functions to model and solve problems.



## Overview of Competency 5

- Understand linear functions through multiple representations (models, tables, graphs, symbols)
- Connect linear functions to proportions and direct variation
- Determine the linear function that best models a data set
- Analyze relationships between linear equations and their graphs
- Model problems using linear functions, inequalities, and systems
- Solve systems of linear equations and inequalities using various methods
- Recognize characteristics, advantages, and limitations of linear models



# Linear Functions

## Sample Question for Competency 5



A water tank contains 500 gallons of water and is being drained at a constant rate of 25 gallons per minute. Which statement about this situation is true?

- A. The relationship is a direct variation because the amount of water varies with time.
- B. The relationship is proportional because it can be modeled with a linear function.
- C. The relationship is linear but not proportional because the  $y$ -intercept is not zero.
- D. The relationship is nonlinear because the water level decreases over time.



# Linear Functions

**Answer: C**



This situation can be modeled by the linear function  $W = 500 - 25t$ , where  $W$  is the water remaining (in gallons) and  $t$  is time (in minutes).

The relationship is linear because:

- It has a constant rate of change (-25 gallons per minute)
- The graph would be a straight line

However, it is NOT proportional because:

- A proportional relationship must pass through the origin (0,0)
- This relationship has a y-intercept of 500, not 0
- When  $t = 0$ ,  $W = 500$  (not 0)
- Proportional relationships have the form  $y = kx$ , while this is  $y = -25x + 500$



This demonstrates an important distinction: all proportional relationships are linear, but not all linear relationships are proportional. Direct variation and proportional relationships require a y-intercept of zero.

# Nonlinear Functions

The teacher understands and uses nonlinear functions and relations to model and solve problems.



## Overview of Competency 6



- Investigate roots, vertex, and symmetry of quadratic functions using various methods
- Connect geometric, graphic, numeric, and symbolic representations of quadratic functions
- Analyze and solve problems involving exponential growth and decay
- Understand connections among proportions, inverse variation, and rational functions
- Apply transformations like  $f(x \pm c)$  to graphs of nonlinear functions
- Use properties and graphs of nonlinear functions to model and solve problems
- Solve systems of quadratic equations and inequalities
- Work with polynomial, rational, radical, absolute value, exponential, logarithmic, trigonometric, and piecewise functions

# Nonlinear Functions

## Sample Question for Competency 6



A biology student is studying bacterial growth. The population of bacteria doubles every 3 hours. If the initial population is 200 bacteria, which function best models the population  $P$  after  $t$  hours?

- A.  $P = 200(2)^t$
- B.  $P = 200(2)^{(t/3)}$
- C.  $P = 200 + 2t$
- D.  $P = 200(3)^t$



# Nonlinear Functions

**Answer: B**



This is an exponential growth problem where the population doubles at regular intervals.

The general form for exponential growth is  $P = P_0(b)^{(t/d)}$ , where:

- $P_0$  = initial population = 200
- $b$  = growth factor = 2 (doubling)
- $t$  = time elapsed
- $d$  = time period for one complete growth cycle = 3 hours

Therefore:  $P = 200(2)^{(t/3)}$



# Calculus Foundations

The teacher uses and understands the conceptual foundations of calculus related to topics in middle school mathematics.

## Overview of Competency 7

- Relate middle school math topics to the concept of limit in sequences and series
- Connect average rate of change to slope of secant line and instantaneous rate of change to slope of tangent line
- Relate middle school math topics to area under a curve
- Understand how calculus concepts answer questions about rates of change, areas, volumes, and properties of functions

# Calculus Foundations

## Sample Question for Competency 7



A car travels 120 miles in 2 hours. During the first hour, it travels 50 miles, and during the second hour, it travels 70 miles. A student calculates that the average speed for the entire trip is 60 mph. Which calculus concept is most directly related to this calculation?

- A. The instantaneous rate of change, represented by the derivative of the position function
- B. The average rate of change, represented by the slope of the secant line connecting two points
- C. The area under the curve of the velocity function over the time interval
- D. The limit of a sequence as time approaches infinity



# Calculus Foundations

**Answer: B**



The average speed calculation (total distance  $\div$  total time = 120 miles  $\div$  2 hours = 60 mph) represents the average rate of change of position with respect to time.

In calculus terms:

- If we graph position (distance) versus time, we get a curve
- The average rate of change is the slope of the secant line connecting the starting point  $(0, 0)$  and ending point  $(2, 120)$
- Slope =  $(120 - 0)/(2 - 0) = 60$  mph

**Domain III**



# Geometry and Measurement



## Domain III Competencies

- 8. MEASUREMENT
- 9. GEOMETRY
- 10. 2D AND 3D FIGURES
- 11. TRANSFORMATIONS

### DOMAIN III

4 Competencies



Domain III  
21%

## Geometry and Measurement

This section has 4 competencies. Review the following slides to understand each competency on the test.

# Measurement

The teacher understands measurement as a process.

## Overview of Competency 8

- Selects and uses appropriate units of measurement (e.g., temperature, money, mass, weight, area, capacity, density, percents, speed, acceleration) to quantify, compare and communicate information.
- Develops, justifies and uses conversions within measurement systems.
- Applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.
- Describes the precision of measurement and the effects of error on measurement.
- Applies the Pythagorean theorem, proportional reasoning and right triangle trigonometry to solve measurement problems.

## Sample Question for Competency 8



A rectangular swimming pool has dimensions of 25 meters by 15 meters by 2 meters deep. The pool is being filled with water at a rate of 0.5 cubic meters per minute. Using dimensional analysis, determine how long it will take to fill the pool completely.



- A. 150 minutes
- B. 750 minutes
- C. 1,500 minutes
- D. 75 minutes

# Measurement

## Answer: C

This problem requires using dimensional analysis to solve a measurement problem involving volume and rate.

Step 1: Calculate the volume of the pool

- Volume = length  $\times$  width  $\times$  height
- $V = 25 \text{ m} \times 15 \text{ m} \times 2 \text{ m} = 750 \text{ m}^3$

Step 2: Use dimensional analysis to find the time

- Rate =  $0.5 \text{ m}^3/\text{minute}$
- Time = Volume  $\div$  Rate
- Time =  $750 \text{ m}^3 \div (0.5 \text{ m}^3/\text{minute})$
- Time =  $750 \text{ m}^3 \times (\text{minute}/0.5 \text{ m}^3)$
- Time = 1,500 minutes

# Geometry

The teacher understands the geometric relationships and axiomatic structure of Euclidean geometry.

## Overview of Competency 9

- Understands concepts and properties of points, lines, planes, angles, lengths and distances.
- Analyzes and applies the properties of parallel and perpendicular lines.
- Uses the properties of congruent triangles to explore geometric relationships and prove theorems.
- Describes and justifies geometric constructions made using a compass and straight edge and other appropriate technologies.
- Applies knowledge of the axiomatic structure of Euclidean geometry to justify and prove theorems.

## Sample Question for Competency 9



A geometry student is asked to prove that the base angles of an isosceles triangle are congruent. Which property of congruent triangles would be most useful in constructing this proof?

- A. Side–Side–Side (SSS) congruence
- B. Side–Angle–Side (SAS) congruence
- C. Angle–Side–Angle (ASA) congruence
- D. Angle–Angle–Side (AAS) congruence



## Answer: B



This problem tests understanding of how to use properties of congruent triangles to prove geometric theorems, specifically the Isosceles Triangle Theorem.

Classic proof strategy: Given: Triangle ABC with  $AB \cong AC$  (isosceles triangle)

Prove:  $\angle B \cong \angle C$  (base angles are congruent)



The teacher analyzes the properties of two- and three-dimensional figures.



## Overview of Competency 10



- Uses and understands the development of formulas to find lengths, perimeters, areas and volumes of basic geometric figures.
- Applies relationships among similar figures, scale and proportion and analyzes how changes in scale affect area and volume measurements.
- Uses a variety of representations (e.g., numeric, verbal, graphic, symbolic) to analyze and solve problems involving two- and three-dimensional figures such as circles, triangles, polygons, cylinders, prisms and spheres.
- Analyzes the relationship among three-dimensional figures and related two-dimensional representations (e.g., projections, cross-sections, nets) and uses these representations to solve problems.

## Sample Question for Competency 10

A cylindrical water tank has a radius of 3 feet and a height of 8 feet. If the dimensions are doubled (radius = 6 feet, height = 16 feet), how does the volume of the larger tank compare to the volume of the original tank?

- A. The volume is 2 times larger
- B. The volume is 4 times larger
- C. The volume is 6 times larger
- D. The volume is 8 times larger



# Geometry

**Answer: D**



Original cylinder:

- Radius ( $r$ ) = 3 feet
- Height ( $h$ ) = 8 feet
- Volume =  $\pi r^2 h = \pi(3)^2(8) = \pi(9)(8) = 72\pi$  cubic feet



Larger cylinder (all dimensions doubled):

- Radius ( $r$ ) = 6 feet
- Height ( $h$ ) = 16 feet
- Volume =  $\pi r^2 h = \pi(6)^2(16) = \pi(36)(16) = 576\pi$  cubic feet

Comparison:

- Ratio =  $576\pi \div 72\pi = 8$
- The larger tank holds 8 times as much water

# Geometry

The teacher understands transformational geometry and relates algebra to geometry and trigonometry using the Cartesian coordinate system.

## Overview of Competency 11

- Describes and justifies geometric constructions made using a reflection device.
- Uses translations, reflections, glide-reflections and rotations to demonstrate congruence
- Uses dilations (expansions and contractions) to illustrate similar figures and proportionality.
- Uses symmetry to describe tessellations and shows how they can be used to illustrate geometric concepts, properties and relationships.
- Applies concepts and properties of slope, midpoint, parallelism and distance in the coordinate plane to explore properties of geometric figures and solve problems.
- Applies transformations in the coordinate plane.
- Uses the unit circle in the coordinate plane to explore properties of trigonometric functions.

## Sample Question for Competency 11

Triangle ABC has vertices at  $A(2, 3)$ ,  $B(5, 3)$ , and  $C(2, 7)$ . The triangle is reflected across the  $y$ -axis to create triangle  $A'B'C'$ , and then translated 4 units down to create triangle  $A''B''C''$ . What are the coordinates of vertex  $C''$ ?

- A.  $(-2, 3)$
- B.  $(-2, 7)$
- C.  $(2, 3)$
- D.  $(-2, -1)$



## Answer: A



Step 1: Reflect triangle ABC across the  $y$ -axis

- Reflection across the  $y$ -axis rule:  $(x, y) \rightarrow (-x, y)$
- Original point  $C(2, 7)$
- After reflection:  $C'(-2, 7)$

Step 2: Translate 4 units down

- Translation down rule:  $(x, y) \rightarrow (x, y - 4)$
- Point  $C'(-2, 7)$
- After translation:  $C''(-2, 7 - 4) = C''(-2, 3)$



Domain IV



# Probability and Statistics



## Domain IV Competencies

12.

DATA ANALYSIS

13.

PROBABILITY

14.

STATISTICS

Domain IV  
16%

## DOMAIN IV

3 Competencies

## Probability and Statistics

This section has 3 competencies. Review the following slides to understand each competency on the test.

# Data Analysis

The teacher understands how to use graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.

## Overview of Competency 12

- Organizes and displays data in a variety of formats (e.g., tables, frequency distributions, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).
- Applies concepts of center, spread, shape and skewness to describe a data distribution.
- Supports arguments, makes predictions and draws conclusions using summary statistics and graphs to analyze and interpret one-variable data.
- Demonstrates an understanding of measures of central tendency (e.g., mean, median, mode) and dispersion (e.g., range, interquartile range, variance, standard deviation).
- Analyzes connections among concepts of center and spread, data clusters and gaps, data outliers and measures of central tendency and dispersion.
- Calculates and interprets percentiles and quartiles.

## Sample Question for Competency 12



A teacher records the test scores for a class of 20 students: 72, 75, 78, 78, 80, 82, 82, 85, 85, 85, 88, 88, 90, 90, 92, 92, 95, 95, 98, 100

The teacher calculates the mean as 86 and the median as 86.5. A student who was absent takes the test and scores 45. How will this new score affect the mean and median?

- A. Both the mean and median will decrease significantly
- B. The mean will decrease significantly, but the median will decrease only slightly
- C. The mean will decrease slightly, but the median will remain unchanged
- D. Both the mean and median will remain approximately the same



## Answer: B

The mean is sensitive to outliers because it uses all values in its calculation, while the median is resistant to outliers because it only depends on the middle value(s). The score of 45 is an outlier that significantly pulls down the mean but has minimal effect on the median. This demonstrates understanding of measures of central tendency, how data outliers affect these measures, and the relative advantages of using median versus mean in the presence of extreme values.

# Probability

The teacher understands the theory of probability.



## Overview of Competency 13

- Explores concepts of probability through data collection, experiments and simulations.
- Uses the concepts and principles of probability to describe the outcome of simple and compound events.
- Generates, simulates and uses probability models to represent a situation.
- Determines probabilities by constructing sample spaces to model situations.
- Solves a variety of probability problems using combinations, permutations and geometric probability (i.e., probability as the ratio of two areas).
- Uses the binomial, geometric and normal distributions to solve problems.



## Sample Question for Competency 13



A bag contains 5 red marbles, 3 blue marbles, and 2 green marbles. If two marbles are drawn from the bag without replacement, what is the probability that both marbles are red?

- A.  $\frac{1}{4}$
- B.  $\frac{2}{9}$
- C.  $\frac{5}{18}$
- D.  $\frac{25}{100}$



**Answer: B**

Using the multiplication rule for dependent events:  $P(\text{both red}) = P(\text{first red}) \times P(\text{second red} \mid \text{first red})$   
 $P(\text{both red}) = (5/10) \times (4/9)$   
 $P(\text{both red}) = 20/90$   
 $P(\text{both red}) = 2/9$

The teacher understands the relationship among probability theory, sampling and statistical inference and how statistical inference is used in making and evaluating predictions.

## Overview of Competency 14

- Applies knowledge of designing, conducting, analyzing and interpreting statistical experiments to investigate real-world problems.
- Demonstrates an understanding of random samples, sample statistics and the relationship between sample size and confidence intervals.
- Applies knowledge of the use of probability to make observations and draw conclusions from single variable data and to describe the level of confidence in the conclusion.
- Makes inferences about a population using binomial, normal and geometric distributions.
- Demonstrates an understanding of the use of techniques such as scatter plots, regression lines, correlation coefficients and residual analysis to explore bivariate data and to make and evaluate predictions.



## Sample Question for Competency 14

A researcher wants to determine if a new teaching method improves student test scores. She randomly assigns 30 students to use the new method and 30 students to use the traditional method. The new method group has a mean score of 85 with a standard deviation of 8, while the traditional method group has a mean score of 78 with a standard deviation of 10. Which statement best describes the appropriate next step in analyzing these results?



- A. Conclude that the new method is better because  $85 > 78$
- B. Calculate the difference ( $85 - 78 = 7$  points) and report this as the improvement
- C. Conduct a statistical test to determine if the difference is statistically significant or could be due to random variation
- D. Increase the sample size to 100 students per group before drawing any conclusions



## Answer: C



- The observed difference (7 points) could be due to the teaching method OR random variation
- Statistical inference uses hypothesis testing to determine if results are statistically significant
- Even with random assignment, samples can differ by chance alone
- A statistical test (like a t-test) helps determine the probability that the observed difference occurred by chance
- The researcher needs to establish a confidence level before making claims about the effectiveness



**Domain V**



# **Mathematical Processes and Perspectives**



## Domain V Competencies

**15.**

REASONING AND PROBLEM SOLVING

**16.**

CONNECTIONS AND COMMUNICATION

Domain V  
10%



**DOMAIN V**  
2 Competencies

## Mathematical Processes and Perspectives

This section has 2 competencies. Review the following slides to understand each competency on the test.

# Reasoning and Problem Solving

The teacher understands mathematical reasoning and problem solving.



## Overview of Competency 15

- Understand proof techniques and use inductive/deductive reasoning to make and evaluate conjectures
- Recognize multiple solution strategies and select appropriate approaches for given problems
- Develop mathematical models of real-world situations and evaluate their effectiveness
- Evaluate the reasonableness of solutions and understand appropriate uses of estimation



## Sample Question for Competency 15



A student claims: "The sum of any two odd numbers is always even." Which approach would best help the student prove this conjecture?

- A. Test several examples like  $3 + 5 = 8$ ,  $7 + 9 = 16$ , and  $11 + 13 = 24$  to show it's true
- B. Use deductive reasoning with algebraic representation: let odd numbers be  $(2n + 1)$  and  $(2m + 1)$ , then show their sum equals  $2(n + m + 1)$ , which is even
- C. Create a counterexample by finding two odd numbers whose sum is odd
- D. Use a calculator to verify the pattern holds for all odd numbers up to 1,000

# Analysis and Response

**Answer: B**

Deductive reasoning provides a formal proof that works for ALL cases:

- Let the first odd number be represented as  $2n + 1$  (where  $n$  is any integer)
- Let the second odd number be represented as  $2m + 1$  (where  $m$  is any integer)
- Sum =  $(2n + 1) + (2m + 1) = 2n + 2m + 2 = 2(n + m + 1)$
- Since  $2(n + m + 1)$  has 2 as a factor, it must be even
- This proves the statement is true for ANY two odd numbers

# Reasoning and Problem Solving

The teacher understands mathematical connections within and outside of mathematics and how to communicate mathematical ideas and concepts.

## Overview of Competency 16



- Recognize and use multiple representations of mathematical concepts (numeric, verbal, graphic, symbolic, concrete)
- Use mathematics to model and solve problems in other disciplines (art, music, science, business)
- Communicate mathematical ideas precisely using appropriate language and visual media
- Apply financial literacy concepts (income, taxes, budgeting, savings) to manage resources effectively



# Reasoning and Problem Solving

## Sample Question for Competency 16



A teacher wants students to understand the concept of slope in multiple contexts. Which set of representations best demonstrates mathematical connections across different disciplines and representations?

A. Show the formula  $m = (y_2 - y_1)/(x_2 - x_1)$ , calculate slope from two points, and graph several lines with different slopes

B. Have students calculate the pitch of a roof (rise over run) in architecture, analyze the steepness of a hiking trail using a topographic map, graph speed vs. time to find acceleration in physics, and represent slope as a rate of change in a data table

C. Complete 20 practice problems finding slope from graphs, tables, and equations

D. Define slope verbally as "the measure of steepness" and show examples on coordinate planes



# Reasoning and Problem Solving

## Answer: B



- Students understand concepts more deeply when they see the same idea in multiple contexts
- Connections to other disciplines make mathematics more relevant and meaningful
- Multiple representations (numeric, verbal, graphic, symbolic, concrete) help students build flexible understanding
- Real-world applications demonstrate the utility of mathematical concepts



**Domain VI**



# Mathematical Learning, Instruction, & Assessment



## Domain V Competencies

**17.** REASONING AND PROBLEM SOLVING

**18.** CONNECTIONS AND COMMUNICATION

**19.** CONNECTIONS AND COMMUNICATION

Domain VI  
16%

**DOMAIN V**  
3 Competencies

## Mathematical Learning, Instruction, & Assessment

This section has 3 competencies. Review the following slides to understand each competency on the test.

# Developing Mathematical Skills

The teacher understands how children learn and develop mathematical skills, procedures and concepts.

## Overview of Competency 17

- Applies principles of learning mathematics to plan appropriate instructional activities for all students.
- Understands how students differ in their approaches to learning mathematics with regard to varied backgrounds.
- Uses students' prior mathematical knowledge to build conceptual links to new knowledge and plans instruction that builds on students' strengths and addresses students' needs.
- Understands how learning may be assisted through the use of mathematics manipulatives and technological tools.
- Understands how to motivate students and actively engage them in the learning process.

# Developing Mathematical Skills

## Sample Question for Competency 17



A teacher is planning a lesson on multiplying fractions. According to research on how students learn mathematics, which instructional sequence would best support conceptual understanding?

A. Show the algorithm (multiply numerators, multiply denominators), then have students practice 20 problems

B. Begin with concrete manipulatives (fraction bars/area models), move to pictorial representations, then introduce the symbolic algorithm

C. Start with the symbolic algorithm, then show one visual example if students seem confused

D. Have students discover the algorithm independently through trial and error



# Developing Mathematical Skills

**Answer: B**



This follows the concrete–pictorial–abstract (CPA) instructional continuum, which is a research–based approach for developing mathematical understanding. Students first manipulate physical models (concrete) to build understanding, then use drawings/diagrams (pictorial) to represent the concept, and finally work with symbols and algorithms (abstract). This progression builds conceptual understanding before procedural fluency, helping students understand why the algorithm works, not just how to execute it. Option A focuses only on procedures without conceptual foundation. Option C reverses the effective sequence. Option D lacks the guided instruction needed for most learners.



# Knowledge of Students and Curriculum

The teacher understands how to plan, organize and implement instruction using knowledge of students, subject matter and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) to teach all students to use mathematics.

## Overview of Competency 18

- Develops clear learning goals to plan, deliver, assess and reevaluate instruction based on the TEKS.
- Understands procedures for developing instruction that establishes transitions between concrete, symbolic and abstract representations of mathematical knowledge.
- Understands how to create a learning environment that provides all students, including English-language learners, with opportunities to develop mathematical skills.
- Demonstrates an understanding of a variety of questioning strategies to help students analyze and evaluate their mathematical thinking.
- Understands how technological tools and manipulatives can be used appropriately to assist students in developing, comprehending and applying mathematical concepts.
- Understands how to relate mathematics to students' lives and a variety of careers and professions.

## Sample Question for Competency 18



A teacher is planning a unit on linear equations aligned with TEKS standards. The class includes several English language learners. Which instructional approach best addresses both content goals and student diversity?

- A. Teach the same lesson to all students using lecture and textbook examples
- B. Use multiple representations (graphs, tables, equations, real-world scenarios), incorporate visual supports, provide sentence frames for mathematical discussions, and use both small-group and whole-class instruction
- C. Provide English language learners with translated worksheets while teaching the rest of the class normally
- D. Simplify the content for English language learners by focusing only on basic skills



# Knowledge of Students and Curriculum

## Answer: B

Effective mathematics instruction uses varied instructional methods and representations to make content accessible to all learners. Multiple representations help all students (not just ELLs) make connections between concepts. Visual supports and sentence frames provide language scaffolding while maintaining high mathematical expectations. Varied grouping (individual, small-group, large-group) allows for differentiation and mathematical discourse. Options A and C don't address diverse learning needs. Option D lowers expectations rather than providing appropriate support, denying ELLs access to grade-level content required by TEKS.



# Mathematical Assessments

The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.

## Overview of Competency 19



- Demonstrates an understanding of the purpose of various assessments in mathematics, including formative and summative assessments.
- Understands how to select and develop assessments that are consistent with what is taught and how it is taught.
- Demonstrates an understanding of how to develop a variety of assessments consisting of worthwhile tasks that assess mathematical understanding, common misconceptions and error patterns.
- Understands how to evaluate a variety of assessment methods and materials for reliability, validity, absence of subjectivity, clarity of language and appropriateness of mathematical level.
- Understands the relationship between assessment and instruction and knows how to evaluate assessment results to design, monitor and modify instruction to improve mathematical learning



# Mathematical Assessments

## Sample Question for Competency 19

After teaching a unit on solving systems of equations, a teacher reviews student work and notices that many students can solve problems using substitution but struggle with elimination method and graphing. What should the teacher do next?

- A. Move on to the next unit since most students mastered at least one method
- B. Re-teach the entire unit from the beginning to all students
- C. Use formative assessment results to provide targeted instruction on elimination and graphing while offering extensions for students who mastered all methods
- D. Give a summative test and assign grades based on current performance



# Mathematical Assessments

## Answer: C

This demonstrates understanding of the relationship between assessment and instruction. Formative assessment (reviewing student work) informs instructional decisions. The teacher should use assessment results to monitor and modify instruction—providing targeted re-teaching for skills not yet mastered (elimination, graphing) while differentiating for students ready to move forward. This approach addresses common misconceptions and error patterns identified through assessment. Option A ignores unmet learning goals. Option B wastes time re-teaching what students already know. Option D treats assessment as evaluation only, missing its instructional purpose of improving student learning.





# Additional Resources

Breakdown of Math 4-8 Test 115

Thank You

TutoringEZ